

HEAT AND MASS TRANSFER IN A TWO-COMPONENT DEVELOPED TURBULENT GAS-VAPOR-DROPLET FLOW

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A calculation model is developed and a numerical study is made of the heat and mass transfer characteristics in a turbulent gas-vapor-droplet flow moving in a round tube. The model takes into account the evaporation of droplets, the diffusion of vapor into air, and the acceleration of a carrier flow. Distributions of the parameters of the two-phase flow are obtained with respect to the tube radius for different initial concentrations of the gas phase. Heat- and mass-transfer calculations are compared to the experimental and numerical works. On the whole, the evaporation of the droplets in the vapor-gas flow leads to the intensification of heat transfer as compared to a one-component vapor-droplet flow and single-phase flow of vapor.

A great number of works are devoted to theoretical investigation of heat transfer in two-phase vapor- and gas-droplet flows [1–9]. In those works, laminar and turbulent modes of vapor-droplet flows are most fully studied numerically [5–7]. However, for practical applications the data on a turbulent flow of gas-vapor-droplet mixtures in channels are of importance. Investigations in this field are scarce [7–9].

In [8], based on the asymptotic theory of a turbulent boundary layer, an approximate method of calculation of heat transfer in the supercritical region of a disperse flow is developed that fairly fits the experimental data for smooth and rough surfaces. A similar approach but for the polydisperse composition of droplets was used in [9] for obtaining the relative law of heat transfer and a logarithmic temperature distribution in the boundary layer of a vapor-droplet flow. However, a large number of the assumptions adopted in these works require verification and accordingly a solution of the problem in a more complete form.

The turbulent heat transfer in a vapor-droplet flow is investigated numerically in [6, 7]. The formulation of the problem as a whole is similar to the well-tested approach for the laminar mode of flow, while the turbulent thermal conductivities and the velocities are adopted in accordance with the Deissler model for a single-phase flow [10].

It should be noted that in the majority of the theoretical investigations performed, heat and mass transfer was studied in one-component vapor-droplet flows. The present work is devoted to further development of the calculated studies of the heat transfer and a parametric analysis of the process in turbulent two-component gas-vapor-droplet flows. The presence of the second component in a gas phase (for instance, of air in the mixture with steam) makes the solution of the problem substantially more complicated, since it necessitates simultaneous solution of the equations of energy and diffusion for a vapor-gas mixture. These data are also of interest for practical applications in calculations of two-phase cooling systems for units of power equipment and for facilities of chemical technologies.

Formulation of the Problem. In the present work, consideration is given to the two-dimensional stabilized steady-state flow of a two-phase gas-vapor-droplet flow in a tube with allowance for evaporation of

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liquid droplets. Such a mechanism of the disperse flow mode was adopted in the majority of studies of heat transfer in vapor-droplet flows [6]; we have also employed this approach in the present work. It was assumed that conductive heat transfer caused by direct contact of a droplet with a wall was negligible as compared to the contribution of the convective heat transfer between the vapor-gas flow and the wall; the radiative heat transfer also was not taken into account. In the vapor-gas flow, droplets serve as a distributed heat sink and a vapor source. The mixture gives off heat to liquid droplets, while the vapor generated is heated to the temperature of the main flow and diffuses into the region with a lower vapor content.

In the inlet cross section of the tube, the temperature distribution of the vapor and the droplets is uniform and the vapor can be superheated relative to the saturation temperature at its present partial pressure. The temperature of a particle over its diameter was also assumed to be constant, since according to the estimates made [11] the Biot number is $Bi = \alpha_0 d_{p1} / \lambda_{liq} < 0.1$, where α_0 is the heat-transfer coefficient of a nonevaporating particle.

All particles at the inlet to the tube are of the same size, and their number concentration per unit volume is also constant; moreover, the latter condition is fulfilled for the entire region of the flow. Heat is transferred from the vapor-gas mixture to the droplets only by conduction. The presence of the droplets does not influence the radial distribution of the flow velocity and of the turbulent thermal conductivity.

Consideration is given to two types of boundary conditions on the inner surface of the tube, namely, the regimes with a constant specific heat flux on the wall ($q_w = \text{const}$) and with a constant wall temperature ($T_w = \text{const}$). In the present work, we have mainly investigated the case $q_w = \text{const}$.

With account for the assumptions made, the heat transfer in a gas-vapor-droplet flow for the axisymmetric flow pattern is described by the system of the equations of energy [12]

$$\rho_m C_{pm} \bar{U} U(r) \frac{\partial T}{\partial x} = \frac{\lambda_m(r)}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - a \pi n d_p^2 (T - T_{liq}) + \rho_m D(r) \frac{\partial K_v}{\partial r} (C_{pv} - C_{pa}) \frac{\partial T}{\partial r} \quad (1)$$

and of diffusion for a vapor-gas mixture [12]

$$\rho_m \bar{U} U(r) \frac{\partial K_v}{\partial x} = \frac{\rho_m D(r)}{r} \frac{\partial}{\partial r} \left(r \frac{\partial K_v}{\partial r} \right) + j_s n d_p^2 \pi, \quad (2)$$

here $D(r)$ is the coefficient of turbulent diffusion of vapor into air, whose value, similarly to the turbulent thermal conductivity, is not constant over the tube radius.

The equations of energy and diffusion have source (sink) terms, which describe heat removal from a gas phase and delivery of vapor mass due to particle evaporation. They are represented by the second terms on the right-hand side of Eqs. (1) and (2). Moreover, the equation of energy (1) contains a term on the right-hand side that is attributable to diffusional heat transfer in the vapor-gas phase.

Relations (1) and (2) are supplemented by the equation of heat transfer at the droplet-vapor-gas mixture interface:

$$C_{p,liq} \rho_{liq} \frac{\pi d_p^2}{6} \frac{\partial T_{liq}}{\partial \tau} = a \pi d_p^2 (T - T_{liq}) - j_s \pi d_p^2 [L + C_{pv} (T - T_{liq})] \quad (3)$$

and by the equation of conservation of vapor mass on the evaporating surface of a droplet:

$$j_s = j_s (K_v)_s - \rho_v D_v \left(\frac{\partial K_v}{\partial r} \right)_s. \quad (4)$$

The quantity α in (3), according to the data of [13], is related to the heat transfer coefficient α_0 by the following relation:

$$\alpha = \frac{\alpha_0}{1 + C_{pm}(T - T_{liq})/L}. \quad (5)$$

The coefficient of heat transfer to finely disperse nonevaporating particles in the absence of a phase slip is described by the relation $Nu = \alpha_0 d_p / \lambda_m = 2$ and, consequently, $\alpha_0 = 2\lambda_m / d_p$.

Considering that the diffusional Stanton number St_d is determined as

$$St_d = -\rho_v D_v \left(\frac{\partial K_v}{\partial r} \right)_s / \rho_m U [(K_v)_s - K_v], \quad (6)$$

we can, with account for (7), write the equation of mass conservation (4):

$$j_s = St_d \rho_m U b_{1d}, \quad (7)$$

where

$$b_{1d} = \frac{(K_v)_s - K_v}{1 - (K_v)_s}. \quad (8)$$

For finely disperse particles in the case of the absence of a phase slip, the mass transfer between the droplets and the mixture is described by the known relations [11]: $Sh = \beta d_p / D_v = 2$ and $St_d = Sh / Re_p Sc = 2 / Re_p Sc$. Then Eq. (7) is transformed finally to the form

$$j_s = \frac{2D_v \rho_m b_{1d}}{d_p}, \quad (9)$$

and the penetrability parameter b_{1d} is determined from Eq. (8) with the use of the saturation curve.

The equation of material balance for the binary vapor-air mixture is as follows:

$$K_v + K_a = 1. \quad (10)$$

For the ternary vapor-gas-liquid mixture it is written as

$$M_v + M_a + M_{liq} = 1. \quad (11)$$

The relation between the mass concentrations K_i and M_i can be written in the following form:

$$K_v = \frac{M_v}{M_v + M_a}; \quad K_a = \frac{M_a}{M_v + M_a} = 1 - K_v. \quad (12)$$

The expression for calculating the running diameter of a droplet d_p is

$$d_p = \sqrt[3]{\frac{6C_{liq} \rho_m}{\pi \rho_{liq} n}}. \quad (13)$$

Thus, relations (1) through (13) represent a close system of equations that describes the processes of heat and mass transfer in a droplet flow and allows calculation of all the sought quantities, namely, the distributions of the temperatures, the enthalpies, the phase components, and the components of the vapor-gas

mixture, and makes it possible to track the dynamics of change in the particle sizes and to analyze the degree of the intensification of heat transfer due to the evaporation processes.

Boundary conditions for the temperatures and the concentration of the components of the vapor-gas mixture are written in the following form:

$$\frac{\partial T}{\partial r} = \frac{\partial K_v}{\partial r} = 0, \quad r = 0, \quad (14)$$

$$\lambda_m \frac{\partial T}{\partial r} = q_w \quad \text{and} \quad \frac{\partial K_v}{\partial r} = 0, \quad r = R. \quad (15)$$

The temperature of the vapor-gas mixture and of the particles at the inlet as well as the concentrations of the vapor, the gas, and the droplets were assumed to be constant over the cross section:

$$T = T_1, \quad T_{\text{liq}} = T_{\text{liq}1}, \quad d_p = d_{p1}, \quad K_v = K_{v1}, \quad K_a = 1 - K_{v1}, \quad M_{\text{liq}} = M_{\text{liq}1} \quad \text{at} \quad x = 0. \quad (16)$$

The local Nusselt number Nu for a constant specific flux on the wall was determined from the wall-temperature gradient and the mass-mean value in the vapor-gas mixture

$$\text{Nu} = \frac{2q_w R}{\lambda_m (T_w - \bar{T})}, \quad (17)$$

where \bar{T} was determined by integration of the temperature field over the tube cross section:

$$\bar{T} = \frac{2}{R^2} \int_0^R T U(r) r dr. \quad (18)$$

Similarly, the mass-mean concentrations of the components of the gas and liquid phases were calculated.

In the case of intense evaporation, the flow rate of the gas phase due to vaporization increases as the flow advances in the tube. Here, it was assumed that the relative velocity profile remained constant but its mass-mean value changed in accordance with the balance relation

$$\bar{U} = \frac{\bar{U}_1}{1 - 4n\rho_{\text{liq}}(d_{p1}^3 - d_p^3)/\rho_m}. \quad (19)$$

In the calculations, we assumed that the diameter of the particles was constant with respect to the radius. Such an assumption is justified by the fact that in turbulent flow owing to the pulsation motion the droplets undergo intense mixing over the radius. As a result, the size of particles in the liquid is a function of only the longitudinal coordinate.

For the carrier phase, use was made of the two-layer turbulence model of Deissler [10] modified by A. Rane and S. Yao [6, 7] for a two-phase flow. The calculational formulas for the turbulent thermal conductivity and for the velocity profiles are given in [7]. The choice of this model was largely attributable to the necessity of carrying out a comparative analysis of calculation results for a simpler case of the turbulent vapor-droplet flow. In the calculations, we also assumed that the turbulent Prandtl and Schmidt numbers were equal: $\text{Pr}_t = \text{Sc}_t = 0.9$ [14]. The Lewis number was $\text{Le} = 1$.

The thermophysical properties of the components were calculated using the formulas from [15].

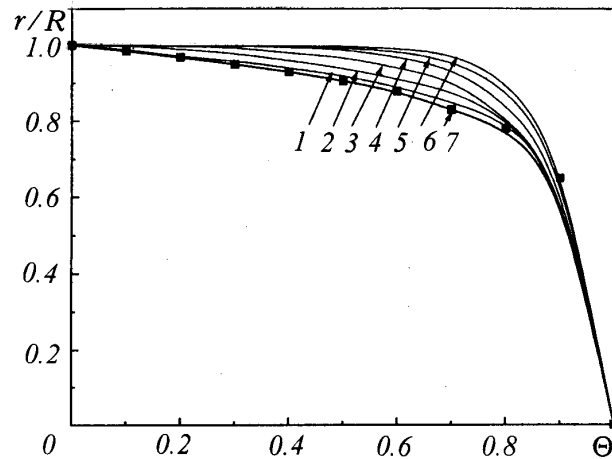


Fig. 1. Influence of the air concentration on the temperature profile of the air-vapor-droplet mixture ($x/(2R) = 20$, $d_{p1} = 30 \mu\text{m}$, $\text{Re} = 10^4$, $q_w = 1 \text{ kW/m}^2$, $T_1 = 373 \text{ K}$, $T_{\text{liq1}} = 283 \text{ K}$, $M_{\text{liq1}} = 0.1$): 1) C_{v1} ; 2) $C_{a1} = 0$ ($C_{v1} = 0.9$); 3) 0.01; 4) 0.1; 5) 0.2; 6) 0.5; 7) $\Theta = (1 - r/R)^{1/7}$.

Algorithm of the Calculations and Verification of Reliability of the Numerical Model. Approximation of the differential equations was carried out by implicit finite-difference analogs with the second order of accuracy in the transverse coordinate and with the first order of accuracy in the longitudinal one. The obtained system of difference equations was solved by the method of elimination by the Thomas algorithm [16]. The tube length was 2 m; its inside diameter was 0.02 m. In the calculations, we used a grid variable relative to the transverse coordinate; in the near-wall zone, where the calculated parameters experience more abrupt changes, the grid was more bunched than in the turbulent core of the flow. In the laminar sublayer the number of points was 10, while in the flow core it was 50. Since the fundamental equations of the mathematical model were nonlinear and conjugate, in obtaining a convergent solution we performed iterations at each calculated point.

In the absence of the liquid phase and vapor, the numerical solution, with an error of no more than 3%, corresponded to the regularities of heat transfer in a stabilized one-phase flow [12]. For the sake of comparison, in the case of the two-phase flow pattern we used the data of a numerical analysis from [6]. The calculations according to the present model were found to be in good agreement with the numerical calculations for a stabilized vapor-droplet flow [6].

Calculation Results and Discussion. Comparison with the Experimental Data. Below we present the results of our investigation of the influence of the parameters of a two-phase flow on heat and mass transfer in a tube. The emphasis has been on studying the effect of the concentration of the gas on a change in the flow characteristics and on the intensification of heat transfer.

All the calculations were carried out for a steam-air mixture at atmospheric pressure with the liquid water particles being present in it. The initial parameters ranged as follows: the temperature of the vapor-gas mixture at the inlet from 373 to 450 K; the flow Reynolds number from $5 \cdot 10^3$ to $1 \cdot 10^6$; the droplet diameter from 1 to 100 μm , their mass concentration from 0 to 0.1, and the concentration of air from 0 to 0.8. As a result of the calculations, we determined the temperatures of the droplets and of the vapor-air mixture, the mass concentrations of all the components, the droplet diameter, and heat transfer to the tube surface.

Calculation results in the form of the dimensionless temperature profiles $\Theta = (T - T_w)/(T_{\text{ax}} - T_w)$ over the tube cross section for different mass concentrations of air are presented in Fig. 1. The fixed quantities in these calculations were the Reynolds number determined from the parameters at the inlet and the concentration of the liquid phase. Curve 1 in this figure represents the temperature profile for the purely single-phase

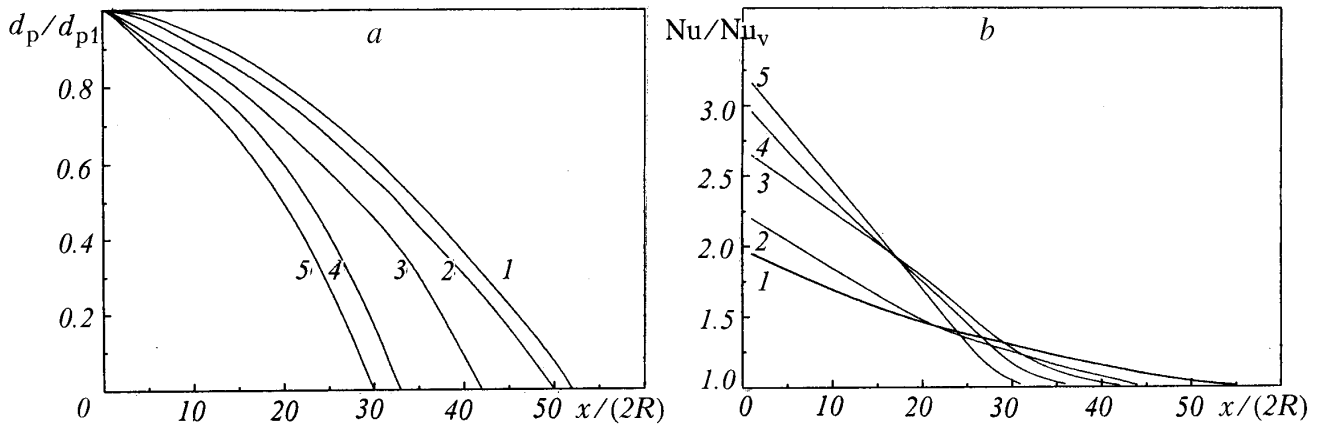


Fig. 2. Change in the droplet diameter (a) and heat transfer (b) in the gas-vapor-droplet flow (the conditions correspond to the data in Fig. 1): 1) $C_{a1} = 0$; 2) 0.01; 3) 0.1; 4) 0.2; 5) 0.5.

mode of the vapor flow $C_{v1} = 1$, while curve 2 pertains to the vapor-droplet flow in the absence of air in it $C_{a1} = 0$. Curve 7, which practically coincides with the profile for the single-phase flow, reflects calculation by the analytical dependence for a single-phase developed turbulent flow [12]. As is seen in Fig. 1, the increase in the content of air at the inlet results in a more filled temperature profile, which causes intensification of the processes of heat transfer to the tube surface. A similar picture, as the calculated studies have revealed, also occurs for the other flow rate and concentration relations at the tube inlet.

The larger filling of the temperature profile is attributable, first of all, to more active evaporation processes between the droplets and the vapor-gas mixture with a high content of air. Indeed, as the air concentration increases, the diffusional transfer of vapor from the surface of particles to the surrounding flow increases, thus causing an increase in the evaporation rate of the droplets. These conclusions are confirmed by the data in Fig. 2a that provides the results of calculation of a change in the droplet sizes over the tube length with variation of the air content. According to these data, droplets evaporate more intensely with increase in the air content. In this case, the length of the two-phase zone decreases.

The distinctive features indicated above are reflected on the parameter of intensification of heat transfer Nu/Nu_v , (Nu_v is the Nusselt number in the single-phase vapor flow at the same Reynolds number). Results of similar calculations are presented in Fig. 2b. For the one-component vapor-droplet flow ($C_{a1} = 0$, curve 1) heat transfer displays the lowest intensification. With increase in the air content, the intensity of heat transfer markedly increases but the length of the zone of the two-phase flow markedly decreases along the channel.

The influence of the initial velocity of the carrier flow on Nu/Nu_v is shown in Fig. 3. As the Reynolds number increases, heat transfer markedly increases. An increase in the droplet size results in the deterioration of heat transfer. For fine particles with $d_{p1} < 0.1\text{--}0.5 \mu\text{m}$, the degree of intensification becomes a fixed quantity, which is due to passage to the equilibrium mode of evaporation of the liquid droplets, in which the vapor-gas mixture is in the saturated state.

It should be emphasized that the data in Fig. 3 are largely of a demonstrative nature since the process under consideration is a multiparametric one and accordingly the degree of intensification is a function of the large number of thermodynamic parameters. A detailed analysis of their influence is beyond the scope of the present work.

Comparison of the calculated results with the available experimental data turned out to be difficult because of the absence of experimental data on heat and mass transfer in a stabilized disperse flow. We employed the experimental data of [2], which is concerned with an investigation of heat and mass transfer in a turbulent gas-vapor-droplet flow. The tube diameter was 12.95 mm, its length was 889 mm, and the Reynolds

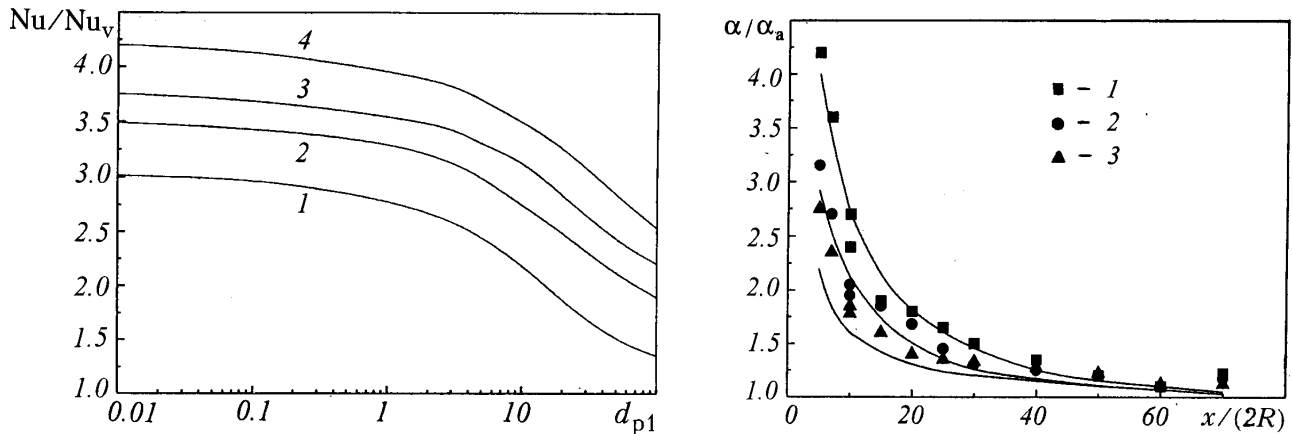


Fig. 3. Intensification of heat transfer as a function of the Reynolds number at the channel inlet ($x/(2R) = 20$, $q_w = 1 \text{ kW/m}^2$, $T_1 = 373 \text{ K}$, $T_{\text{liq}} = 283 \text{ K}$, $M_{\text{liq}} = 0.1$, $M_{\text{a1}} = 0.1$): 1) $\text{Re} = 1.8 \cdot 10^3$; 2) 10^4 ; 3) $5 \cdot 10^4$; 4) 10^5 . d_{p1} μm .

Fig. 4. Comparisons of the experimental and calculated data ($T_1 = 303 \text{ K}$, $M_{\text{liq}} = 0.01$, $\text{Re} = 2.3 \cdot 10^4$, and $d_{p1} = 23 \mu\text{m}$; solid lines, calculation; points, experiment [2]): 1) 6.4 kW/m^2 ; 2) 8.34 ; 3) 14.53 .

number based on the tube diameter was $\text{Re} = 10^4 - 10^6$. The flow pressures ranged from 1.07 to 1.22 bars. The specific heat flux over the wall was $6.4 - 36.2 \text{ kW/m}^2$, and the initial size of the droplets ranged from 8.6 to $23.6 \mu\text{m}$. In the experiment, we measured the wall temperatures in the single-phase flow of air and for the disperse gas-droplet flow of the mixture along the heat-transfer channel. From these values the intensification of heat transfer in the gas-droplet flow was calculated and compared to the single-phase gas flow.

Results of a comparison of the experimental and calculated intensifications of heat transfer are presented in Fig. 4, where α/α_a is the intensification of heat transfer and α_a is the heat-transfer coefficient in the single-phase gas flow.

It follows from the figure that the calculation according to the present model is in fair agreement with the experimental results. The calculated and experimental values of the intensification of heat transfer are characterized by a monotonic decrease along the channel. A more pronounced discrepancy in the calculation results and experimental data at the beginning of the channel is attributed to the influence of the initial portion, on which a dynamic boundary layer develops in experiments. In this zone, the discrepancy of the calculated and experimental results lies in the range 20–25%. On the basic part of the channel the discrepancy does not exceed 10%.

Thus, the developed model, as a whole, qualitatively and quantitatively describes heat and mass transfer in a two-component, two-phase flow in the presence of phase changes. At the same time, it cannot claim completeness and adequacy in describing all the complex features of the occurrence of simultaneous dynamic and heat- and mass-transfer processes; for this, more detailed experimental and numerical studies are needed.

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NOTATION

b_{1d} , diffusion parameter of vapor blast from the evaporating particle; C_p , specific heat at constant pressure, $\text{J}/(\text{kg}\cdot\text{K})$; d_{p1} and d_p , initial and running particle diameter, respectively, m ; $D(r)$, coefficient of tur-

bulent diffusion of vapor into air, m^2/sec ; D_v , coefficient of molecular diffusion of vapor into air, m^2/sec ; j_s , transverse vapor flow over the surface of the evaporating droplet, $\text{kg}/(\text{m}^2 \cdot \text{sec})$; K_v , concentration of the vapor in the binary vapor-air mixture; $(K_v)_s$, mass concentration of the vapor over the particle surface corresponding to the saturation parameters at the droplet temperature T_{liq} ; M , mass concentration of the components in the ternary gas-vapor-droplet flow; L , latent heat of the phase transition, J/kg ; n , number concentration of the liquid droplets in the tube, m^{-3} ; T , temperature, K ; \bar{T} , mass-mean temperature, K ; q_w , specific heat flux over the wall, W/m^2 ; \bar{U} , mean-flow-rate velocity of the flow in the running cross section, m/sec ; $U(r)$, relative radial profile of the flow velocity; U , flow velocity at the calculated point, m/sec ; r , transverse coordinate, m ; x , longitudinal coordinate, m ; R , tube radius, m ; α_0 , heat-transfer coefficient of the nonevaporating particle, $\text{W}/(\text{m}^2 \cdot \text{K})$; α , heat-transfer coefficient in the evaporating droplet, $\text{W}/(\text{m}^2 \cdot \text{K})$; β , mass-transfer coefficient, m/sec ; λ , thermal conductivity, $\text{W}/(\text{m} \cdot \text{K})$; $\lambda(r)$, turbulent thermal conductivity, $\text{W}/(\text{m} \cdot \text{K})$; ρ , density, kg/m^3 ; ν , kinematic viscosity, m^2/sec ; τ , time, sec ; a , thermal diffusivity, m^2/sec . Similarity numbers: $\text{Bi} = \alpha_0 d_{p1} / \lambda_{\text{liq}}$, Biot number; $\text{Re} = \bar{U} 2R / \nu$ and $\text{Re}_p = \bar{U} d_p / \nu$, Reynolds numbers for the tube and for the droplet, respectively; $\text{St}_d = -\rho_v D_v \left[\frac{\partial K_v}{\partial r} \right]_s / \rho_m U [(K_v)_s - K_v]$, diffusional Stanton number; $\text{Sc} = \nu / D$, Schmidt number; $\text{Sh} = \beta d_p / D_v$, Sherwood number; $\text{Pr} = \nu / a$, Prandtl number; $\text{Le} = \text{Pr} / \text{Sc}$, Lewis number; $\text{Nu} = 2q_w R / \lambda_m (T_w - T)$, Nusselt number. Subscripts and superscripts: 0, nonevaporating particle; 1, initial parameter; ax, quantity on the tube axis; d, diffusional parameter; w, wall; liq, liquid phase; p, liquid droplet; a, air; v, vapor; s, parameter under saturation conditions, droplet surface; m, vapor-air mixture; t, turbulent.

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